An Optimal Control Approach to Learning in SIDARTHE Epidemic model

Andrea Zugarini[1,2] Enrico Meloni[1,2] Alessandro Betti[2] Andrea Panizza[2] Marco Corneli[3,4] Marco Gori[2,5]

[1]DINFO, University of Florence, Florence, Italy [2]Department of Information Engineering and Mathematics, University of Siena, Siena, Italy. [3]Université Côte d'Azur Center of Modeling, Simulation & Interaction, Nice, France

[4]Inria, CNRS, Laboratoire J.A. Dieudonné, Maasai research team, Nice, France [5]Inria, CNRS, I3S, Maasai, Université Côte d'Azur, Côte d'Azur, France

Pre-print

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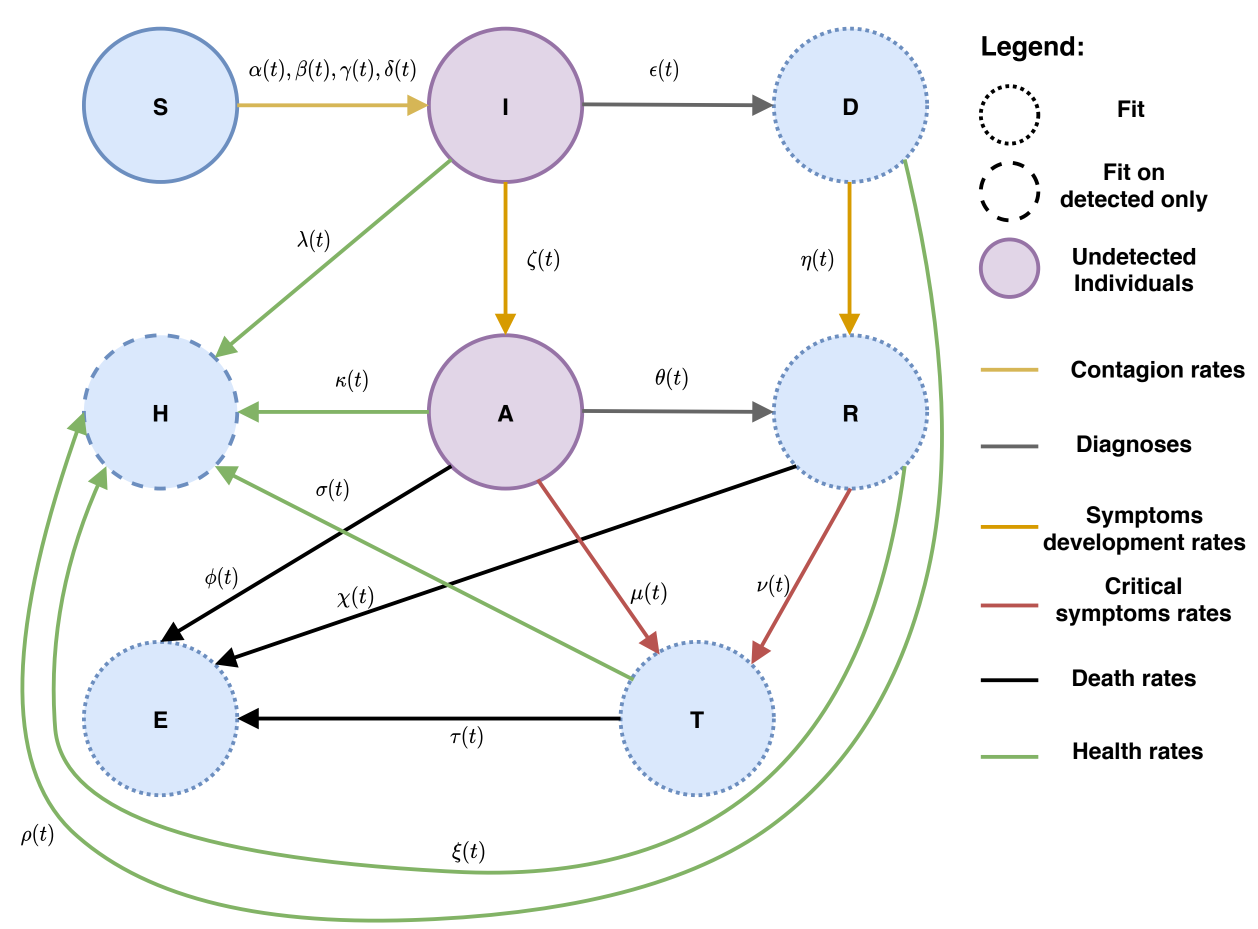
Context

We focus on the problem of learning **time-variant coefficients** of the SIDARTHE [1] epidemic model. We forecast the spread of COVID-19 outbreak in Italy and France. The problem is formulated in terms of a **functional risk** that depends on the learning variables through the solutions of a **dynamic system**. The resulting **variational problem** is then solved by using a **gradient flow** on a suitable, **regularized** functional.

SIDARTHE



$S(t)$ susceptible individuals, $I(t)$ asymptomatic infected undetected, $D(t)$ asymptomatic infected detected, $A(t)$ symptomatic infected undetected, $R(t)$ symptomatic infected detected, $T(t)$ acutely symptomatic infected detected, $H(t)$ healed, $E(t)$ deceased.

Differential Equations

$$\begin{cases}
\dot S(t) = -S(t)\big(\alpha I(t) + \beta D(t) + \gamma A(t) + \delta R(t)\big);\\
\dot I(t) = S(t)\big(\alpha I(t) + \beta D(t) + \gamma A(t) + \delta R(t)\big) - (\varepsilon + \zeta + \lambda) I(t);\\
\dot D(t) = \varepsilon I(t) - (\eta + \rho) D(t);\\
\dot A(t) = \zeta I(t) - (\theta + \mu + \kappa + \phi) A(t);\\
\dot R(t) = \eta D(t) + \theta A(t) - (\nu + \xi + \chi) R(t);\\
\dot T(t) = \mu A(t) + \nu R(t) - (\sigma + \tau) T(t);\\
\dot H(t) = \lambda I(t) + \rho D(t) + \kappa A(t) + \xi R(t) + \sigma T(t);\\
\dot E(t) = \phi A(t) + \chi R(t) + \tau T(t),
\end{cases}$$

with initial conditions:

$$(S^0, I^0, D^0, A^0, R^0, T^0, H^0, E^0) =: z_0,$$

and parameters:

$$\begin{aligned} u(t) =&(\alpha(t), \beta(t), \gamma(t), \delta(t), \varepsilon(t), \zeta(t), \eta(t), \theta(t), \kappa(t),\\ &\lambda(t), \mu(t), \nu(t), \xi(t), \rho(t), \sigma(t), \phi(t), \chi(t), \tau(t)).\end{aligned}$$

Reproduction Number

$$R_0 := \frac{1}{\varepsilon + \xi}\left(\alpha + \frac{\beta\varepsilon}{\eta + \rho} + \frac{\gamma\zeta}{\theta + \mu + \kappa + \phi} + \frac{\delta}{\nu + \xi + \chi}\left(\frac{\eta\varepsilon}{\eta + \rho} + \frac{\zeta\theta}{\theta + \mu + \kappa}\right)\right).$$

Learning SIDARTHE Coefficients

The parameters u are learned by fitting the corresponding compartments $\overline{D}, \overline{R}, \overline{T}, \overline{H}, \overline{E}$ to the official data $\hat D, \hat R, \hat T, \hat H_d, \hat E$. We minimize the functional risk $F(u)$ defined below:

$$F(u) := \int_0^T \frac{m}{2}|\dot u(t)|^2 + \varphi(t, u)\, dt, \qquad m > 0$$

$$\begin{aligned}\varphi(t,u) :=& \frac{e_D}{2}(\overline{D}(t,u,z_0) - \hat D(t))^2 + \frac{e_R}{2}(\overline{R}(t,u,z_0) - \hat R(t))^2\\ &+ \frac{e_T}{2}(\overline{T}(t,u,z_0) - \hat T(t))^2 + \frac{e_H}{2}(\overline{H}_d(t,u,z_0) - \hat H(t))^2\\ &+ \frac{e_E}{2}(\overline{E}(t,u,z_0) - \hat E(t))^2.\end{aligned}$$

Data

Italian and French epidemiological data, gathered from official daily reports up to September 30, 2020.

Data was split into train, validation and test.

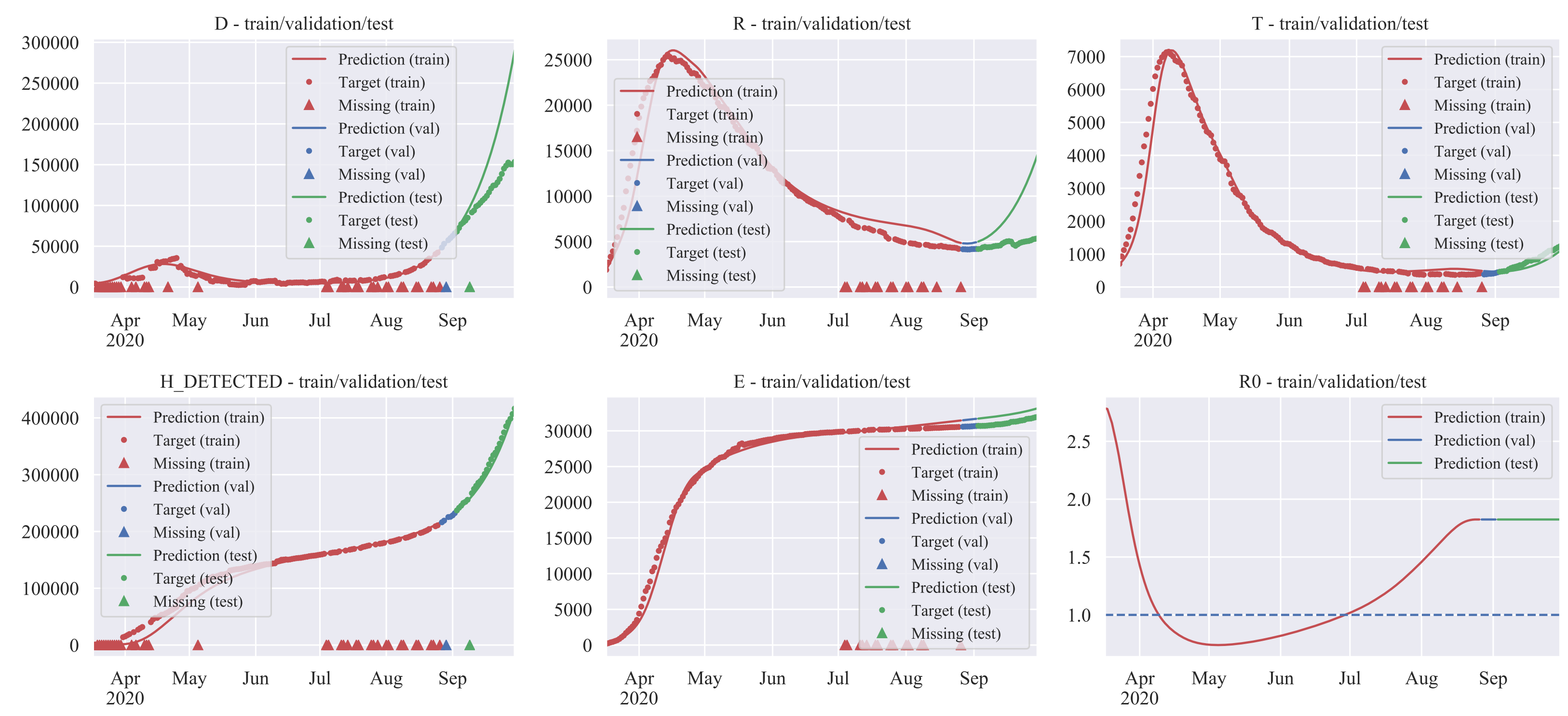
Outbreak forecasting

Model forecast on Italian and French Test data. Mean Absolute Percentage Error (MAPE), and the fraction of days d where the predictions are within an error threshold of 30%.

	Italy		France	
	MAPE	d	MAPE	d
D	16%	20/25	41%	10/25
R	8%	25/25	84%	3/25
T	19%	25/25	16%	25/25
H	4%	25/25	2%	24/24
E	6%	25/25	5%	25/25

Italy

France



References

[1] G. Giordano, F. Blanchini, R. Bruno, P. Colaneri, A. Di Filippo, A. Di Matteo, and M. Colaneri. Modelling the covid-19 epidemic and implementation of population-wide interventions in italy. *Nature Medicine*, Apr 2020.